Free Response Questions

1. (C) Grass clippings are placed in a bin, where they decompose. For  , the amount of grass clippings remaining in the bin is modeled by  , where  is measured in pounds and *t* is measured in days.
   1. Find the average rate of change of  over the interval . Indicate units of measure.
   2. Find the value of . Using correct units, interpret the meaning of the value in the context of the problem.
   3. Find the time *t* for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bind over the interval .
   4. For , , the linear approximation to *A* at , is a better model for the amount of grass clippings remaining in the bin. Use  to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *t*  (minutes) | 0 | 2 | 5 | 8 | 12 |
| (meters/minute) | 0 | 100 | 40 | -120 | -150 |

1. (NC) Train *A* runs back and forth on an east-west section of railroad track. Train *A*’s velocity, measured in meters per minute, is given by a differentiable function , where time *t* is measured in minutes. Selected values for  are given in the table above.
   1. Find the average acceleration of train *A* over the interval .
   2. Do the data in the table support the conclusion that train *A*’s velocity is -100 meters per minute at some time *t* with ? Give a reason for your answer.
   3. A second train, train *B*, travels north from the Origin Station. At time *t* the velocity of train *B* is given by , and at time  the train is 400 meters north of the station and train A is 300 meters east. Find the rate, in meters per minute, at which the distance between train *A* and train *B* is changing at time .

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | -2 | -2<x<-1 | -1 | -1<x<1 | 1 | 1<x<3 | 3 |
| *f(x)* | 12 | Positive | 8 | Positive | 2 | Positive | 7 |
| *f’(x)* | -5 | Negative | 0 | Negative | 0 | Positive | ½ |
| *g(x)* | -1 | Negative | 0 | Positive | 3 | Positive | 1 |
| *g’(x)* | 2 | Positive | 3/2 | Positive | 0 | Negative | -2 |

1. (NC) The twice-differentiable functions *f* and *g* are defined for all real numbers *x*. Values of *f*, *f’*, *g,* and *g’* for various values of *x* are given in the table above.
   1. Find the *x*-coordinate of each relative minimum of *f* on the interval . Justify your answers.
   2. Explain why there must be a value *c*, for , such that .
   3. The function *h* is defined by . Find . Show the computations that lead to your answer.
2. (NC) Consider the differential equation . Let  be the particular solution to the differential equation with the initial condition . The function *f* is defined for all real numbers. Write an equation for the line tangent to the curve at the point (0,1). Use the equation to approximate .

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *t*  (minutes) | 0 | 12 | 20 | 24 | 40 |
| (meters/minute) | 0 | 200 | 240 | -220 | 150 |

1. (NC) Johanna jogs along a straight path. For , Johanna’s velocity is given by a differentiable function *v*. Selected values of , where *t* is measured in minutes and  is measured in meters per minute, are given in the table above.
   1. Bob is riding his bicycle along the same path. For , Bob’s velocity is modeled by , where *t* is measured in minutes and  is measured in meters per minute. Find Bob’s acceleration at time .
   2. Based on the model *B* from part (a), find Bob’s average velocity during the interval .
2. (NC) Consider the differential equation .
   1. Let be a particular solution to the differential equation with the initial condition . Does *f* have a relative minimum, a relative maximum, or neither at ? Justify your answer.
   2. Find  in terms of *x* and *y*. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

*y*

*x*

*O*

2

1

-3

-2

-1

3

4

Graph of *f’*

1. (NC) The figure above shows the graph of , the derivative of a twice-differentiable function *f*, on the interval . The graph of  has horizontal tangents at , , and . The areas of the regions bounded by the *x*-axis and the graph of  on the intervals  and  are 9 and 12, respectively.
   1. Find all *x*-coordinates at which *f* has a relative maximum. Give a reason for your answer.
   2. On what open interval contained in  is the graph of *f* both concave down and decreasing? Give a reason for your answer.
   3. Find the *x*-coordinates of all points of inflection for the graph of *f*. Give a reason for your answer.
2. (NC) Consider the curve given by the equation . It can be shown that .
   1. Write an equation for the line tangent to the curve at the point .
   2. Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.
   3. Evaluate  at the point on the curve where  and .